Algebra review

The algebra toolkit every MYP 5 unit uses: expanding, factorising and solving two equations at once.

Key ideas

- **Expanding:** multiply every term in one bracket by every term in the other, then collect like terms.
- **Quadratic expansions:** $(a + b)^2 = a^2 + 2ab + b^2$ · $(a - b)^2 = a^2 - 2ab + b^2$ · $(a + b)(a - b) = a^2 - b^2$
- **Factorising:** take out the HCF first. $6x^2 + 9x = 3x(2x + 3)$
- **Quadratics:** $x^2 + bx + c = (x + p)(x + q)$, where $p + q = b$ and $pq = c$
- **Simultaneous equations:** two equations, two unknowns. Solve by substitution or elimination.

How to do it

1. **Double brackets:** first, outer, inner, last, then collect like terms.
2. **Factorise in order:** HCF first, then difference of two squares, then the p, q pair.
3. **Elimination:** match one variable's coefficients, add or subtract, solve, substitute back.
4. **Check:** expand your factorised answer. You should get the original back.

Worked example

Solve $3x + 2y = 16$ and $x - y = 2$

→ From the 2nd: $x = y + 2$

→ $3(y + 2) + 2y = 16$ → $5y = 10$ → $y = 2$

→ $x = 4$. Solution **(4, 2)**

Watch out!

- $(x + 5)^2 \neq x^2 + 25$. It is $x^2 + 10x + 25$.
- A minus outside a bracket changes every sign inside.
- HCF first: $2x^2 - 8 = 2(x + 2)(x - 2)$.

Kong's tips

- Say the signs out loud when you expand: most marks are lost on signs.
- Write simultaneous answers as a pair: (4, 2).

Level 7–8 challenge

Unfamiliar, multi-step problems: show every step and explain your answer.

Q1 Expand and simplify $(2x - 3)^2 - (x + 1)(x - 4)$.

Check your answer: $(4x^2 - 12x + 9) - (x^2 - 3x - 4) = 3x^2 - 9x + 13$.

Q2 3 coffees and 2 muffins cost $17.50. 2 coffees and 3 muffins cost $16.25. Find the price of each. Check your answer.

Check your answer: $3c + 2m = 17.5$ and $2c + 3m = 16.25$. ×3 and ×2, then subtract: $5c = 20$, so $c = \$4$ and $m = \$2.75$. Check: $8 + 8.25 = 16.25$.

1 ♠

Sequences & proportions

Number patterns, formulas that change subject, quantities in proportion, and algebraic fractions.

Key ideas

- **Arithmetic:** add d each time. $u_n = u_1 + (n - 1)d$ and $S_n = n/2\ (2u_1 + (n - 1)d)$
- **Geometric:** multiply by r each time. $u_n = u_1 r^{n-1}$ and $S_n = u_1(r^n - 1) \div (r - 1)$
- **Direct proportion:** $y = kx$ (or kx^2, kx^3). **Inverse:** $y = k/x$ (or k/x^2)
- **Rearranging:** do inverse operations to both sides to change the subject.
- **Rational expressions:** factorise top and bottom, then cancel common FACTORS.

Worked example

3, 6, 12, 24, ... Find u_{10} and S_{10}

→ Ratio r = 2, so it is geometric

→ $u_{10} = 3 \times 2^9 =$ **1536**

→ $S_{10} = 3(2^{10} - 1) \div (2 - 1) =$ **3069**

Watch out!

- The power is n − 1, not n.
- Cancel factors only, never terms: $(x + 3)/3 \neq x + 1$.
- r can be a fraction or negative: 16, 8, 4 has r = ½.

How to do it

1. **Which sequence?** Same difference → arithmetic. Same ratio → geometric.
2. **Write down u_1, d (or r) and n**, then use the formula.
3. **Proportion:** write it with k, find k from a known pair, rewrite, answer.
4. **Simplify:** $(x^2 - 9) \div (x^2 + x - 6) = (x + 3)(x - 3) \div (x + 3)(x - 2) = (x - 3) \div (x - 2)$

Kong's tips

- Arithmetic and geometric sequences are a key focus this year: practise both sum formulas.
- In $y = k/x^2$, doubling x makes y a quarter.

☆ Level 7–8 challenge

Unfamiliar, multi-step problems: show every step and explain your answer.

Q1 Gym A costs $40 in month 1 and rises by $2.50 each month. Gym B costs $40 in month 1 and rises by 5% each month. Which costs more in month 12? Find Gym A's total over 12 months.

Check your answer: A: $40 + 11 \times 2.5 =$ $67.50. B: $40 \times 1.05^{11} \approx$ $68.41, so B costs more. Gym A total: $S_{12} = 6(80 + 27.5) =$ $645.

Q2 Simplify $(2x^2 + 7x + 3)/(x^2 - 9) \div (2x + 1)/(x - 3)$.

Check your answer: $(2x + 1)(x + 3)/((x + 3)(x - 3)) \times (x - 3)/(2x + 1)$. Everything cancels, so the answer is 1 (for $x \neq 3, -3, -½$).

2 ♣

Functions & quadratics

Functions, quadratics in every form, graph transformations and rational functions with asymptotes.

Key ideas

- **Function:** each input has one output. Domain = inputs, range = outputs, written f(x).
- **Vertex form:** $y = a(x - h)^2 + k$ has vertex (h, k). Completing the square gets you there.
- **Discriminant** $\Delta = b^2 - 4ac$: $\Delta > 0$ two roots, $\Delta = 0$ one root, $\Delta < 0$ no real roots.
- **Transformations:** $f(x) + k$ moves up k · $f(x - h)$ moves right h · $af(x)$ stretches by a · $-f(x)$ and $f(-x)$ reflect
- **Rational function** $y = a/(x - h) + k$: asymptotes $x = h$ and $y = k$.

How to do it

1. **Complete the square:** $x^2 + 6x + 2 = (x + 3)^2 - 9 + 2 = (x + 3)^2 - 7$, vertex $(-3, -7)$.
2. **Discriminant problems:** write $b^2 - 4ac$ with k in it, then set it > 0, $= 0$ or < 0.
3. **Rational equations:** multiply every term by the common denominator, solve, then reject values that make a denominator 0.
4. **Combined transformations:** stretches and reflections first, then translations.

Worked example

Solve $6/x - 1 = x$

→ Multiply by x: $6 - x = x^2$

→ $x^2 + x - 6 = 0$ → $(x + 3)(x - 2) = 0$

→ **$x = -3$ or $x = 2$** (both fine, $x \neq 0$)

Watch out!

- $f(x - 3)$ moves RIGHT 3, not left.
- Check rational-equation answers: one that makes a denominator 0 is not a solution.
- Asymptotes are lines: write $x = 2$, not just 2.

Kong's tips

- "Touches" means $\Delta = 0$. "Doesn't meet" means $\Delta < 0$.
- Label intercepts, the vertex and asymptotes on every sketch.

Level 7–8 challenge

Unfamiliar, multi-step problems: show every step and explain your answer.

Q1 For which values of k does $x^2 + kx + 9 = 0$ have two different real solutions?

Check your answer: $\Delta = k^2 - 4(1)(9) = k^2 - 36$. Two roots need $\Delta > 0$, so $k^2 > 36$: $k < -6$ or $k > 6$.

Q2 Describe the transformations that map $y = 1/x$ onto $y = 2/(x - 3) + 1$, and state the asymptotes of the new graph.

Check your answer: Vertical stretch by factor 2, then a translation 3 right and 1 up. Asymptotes: $x = 3$ and $y = 1$.

3 ♥

Trigonometry

Trigonometry beyond right angles: sine and cosine rules, triangle area, 3D problems and trig graphs.

Key ideas

- **Special angles:** sin 30° = ½, cos 30° = √3/2, tan 45° = 1, sin 60° = √3/2, tan 60° = √3
- **Sine rule:** a/sin A = b/sin B = c/sin C
- **Cosine rule:** $a^2 = b^2 + c^2 - 2bc \cos A$
- **Area of a triangle** = ½ab sin C
- **Trig graphs:** y = a sin(bx) + d has amplitude |a|, period 360° ÷ b and centre line y = d.

How to do it

1. **Which rule?** A side–angle pair you know → sine rule. Three sides, or two sides and the angle between → cosine rule.
2. **Angle from the cosine rule:** $\cos A = (b^2 + c^2 - a^2) \div 2bc$
3. **3D problems:** find the right-angled triangle you need, redraw it flat, then use Pythagoras or SOH CAH TOA.
4. **Graphs:** read amplitude, period and centre line straight from the equation.

Worked example

b = 7 cm, c = 9 cm, A = 50°. Find a and the area.

→ $a^2 = 49 + 81 - 126 \cos 50° \approx 49.0$

→ a ≈ **7.00 cm**

→ Area = ½ × 7 × 9 × sin 50° ≈ **24.1 cm²**

Watch out!

- Calculator in DEG mode.
- Finding an angle with the sine rule can give two answers (the ambiguous case).
- In 3D, check the right angle really is where you think.

Kong's tips

- Label each side with the small letter of the angle opposite it.
- Sketch the 3D triangle separately and label every length.

Level 7–8 challenge

Unfamiliar, multi-step problems: show every step and explain your answer.

Q1 Two boats leave a port. One sails 12 km on a bearing of 040°, the other 9 km on a bearing of 150°. How far apart are they, and what is the area of the triangle they make with the port?

Check your answer: Angle at the port = 110°. $d^2 = 144 + 81 - 216 \cos 110° \approx 298.9$, so d ≈ 17.3 km. Area = ½ × 12 × 9 × sin 110° ≈ 50.7 km².

Q2 A box measures 8 cm × 6 cm × 5 cm. Find the length of its space diagonal and the angle it makes with the base.

Check your answer: Base diagonal = $\sqrt{(8^2 + 6^2)} = 10$ cm. Space diagonal = $\sqrt{(10^2 + 5^2)} = \sqrt{125} \approx 11.2$ cm. tan θ = 5/10, so θ ≈ 26.6°.

4 Probability

Sets and Venn diagrams, combined events, and probabilities that depend on what you already know.

Key ideas

- **A ∩ B:** in both. **A ∪ B:** in either. **A':** not in A. Three-set Venns: fill the centre first.
- **Axioms:** $0 \le P(A) \le 1$, all outcomes add to 1, and $P(A') = 1 - P(A)$
- **Addition rule:** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Mutually exclusive → $P(A \cap B) = 0$
- **Multiplication rule (independent):** $P(A \cap B) = P(A) \times P(B)$
- **Conditional:** $P(A \mid B) = P(A \cap B) \div P(B)$

How to do it

1. **Pick a diagram:** Venn for overlapping groups, tree for stages, two-way table for two categories.
2. **Tree diagrams:** multiply along the branches, add between paths.
3. **"Given that":** restrict to that group, so divide by its total.
4. **Independent?** Check whether $P(A) \times P(B) = P(A \cap B)$.

Worked example

60 students: 35 like maths, 25 science, 10 both. P(science | maths)?

→ $P(M \cap S) = 10/60$ and $P(M) = 35/60$

→ $P(S \mid M) = 10 \div 35$

→ = **2/7**

Watch out!

- Mutually exclusive is not the same as independent.
- $P(A \mid B) \neq P(B \mid A)$.
- In a three-set Venn, subtract the centre from each overlap.

Kong's tips

- Write what each letter means before you start.
- Check that every region of a Venn diagram adds up to the total.

Level 7–8 challenge

Unfamiliar, multi-step problems: show every step and explain your answer.

Q1 $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cup B) = 0.7$. Find $P(A \cap B)$. Are A and B independent? Find $P(A \mid B)$.

Check your answer: $P(A \cap B) = 0.5 + 0.4 - 0.7 = 0.2$. $0.5 \times 0.4 = 0.2$, so yes, independent. $P(A \mid B) = 0.2 \div 0.4 = 0.5$.

Q2 Machine X makes 60% of a factory's bolts, and 3% of them are faulty. Machine Y makes 40%, and 5% are faulty. A bolt is faulty. What is the probability it came from Y?

Check your answer: $P(\text{faulty}) = 0.6 \times 0.03 + 0.4 \times 0.05 = 0.038$. $P(Y \mid \text{faulty}) = 0.02 \div 0.038 = 10/19 \approx 0.53$.

5 ♠

Statistics & logarithms

Fair samples, relationships between two variables, and logarithms for solving exponential equations.

Key ideas

- **Sampling:** random (equal chance) · systematic (every kth) · stratified (random within groups, in proportion) · quota (a set number per group, NOT random)
- **Scatter plots:** positive, negative or no correlation; strong or weak. Correlation is not causation.
- **Correlation coefficient r** runs from −1 to 1. The regression line passes through $(\bar{x}, \bar{y})$.
- **Logarithm:** $\log_a x = y$ means $a^y = x$. $\log 100 = 2$, $\log_2 8 = 3$
- **Log laws:** $\log(xy) = \log x + \log y$ · $\log(x/y) = \log x - \log y$ · $\log x^n = n \log x$

Worked example

Solve $3^x = 50$

→ Take logs: $x \log 3 = \log 50$

→ $x = \log 50 \div \log 3$

→ $x \approx$ **3.56**

Watch out!

- Quota sampling is not random: the interviewer chooses who to ask.
- $\log(x + y) \neq \log x + \log y$.
- Extrapolating a regression line is unreliable.

How to do it

1. **Stratified sample:** group size ÷ population × sample size.
2. **Regression line:** predict only inside the data range.
3. **Solve $a^x = b$:** take logs of both sides, so $x = \log b \div \log a$.
4. **Log equations:** combine into one log with the laws, then rewrite in exponent form.

Kong's tips

- Say HOW a sample could be biased, in context.
- Check a log answer by substituting: $3^{3.56} \approx 50$.

Level 7–8 challenge

Unfamiliar, multi-step problems: show every step and explain your answer.

Q1 A colony starts with 500 bacteria and doubles every 3 hours: $N = 500 \times 2^{t/3}$. After how many hours will there be 20 000?

Check your answer: $2^{t/3} = 40$, so $t/3 = \log 40 \div \log 2 \approx 5.32$. $t \approx 16.0$ hours.

Q2 For revision hours x (from 0 to 15) and test score y, the regression line is $y = 2.4x + 31$ and $r = 0.87$. Interpret r, predict the score for 10 hours, and comment on using it for 30 hours.

Check your answer: Strong positive correlation. 10 hours: y = 55. 30 hours is outside the data, so the prediction (103) is unreliable, and correlation doesn't prove cause.