Ratios & proportions

A ratio compares two amounts in the same unit. A proportion says two ratios are equal.

Key ideas

- **Ratio:** 3 : 4 = 3/4 = 0.75 = 75%
- **Simplify** by dividing both parts by the HCF: 12 : 18 = 2 : 3
- **Equivalent ratios** simplify to the same ratio, e.g. 12 : 18 and 6 : 9
- **Proportion:** two equal ratios, e.g. 12/18 = 6/9
- **Proportional relationship:** y = kx, where k is the **constant of proportionality**. Graph = straight line through (0, 0)

How to do it

1. **Share in a ratio:** add the parts, find one part, multiply. $60 in 2 : 3 → 60 ÷ 5 = 12 → $24 and $36
2. **Solve a proportion:** cross-multiply, then divide to get the unknown alone.
3. **Test a table:** work out y ÷ x for every pair. Always the same? It is proportional, and that number is k.

Worked example

Solve 12/14 = 18/x

→ Cross-multiply: 12x = 14 × 18 = 252

→ Divide: x = 252 ÷ 12 = **21**

→ Check: 12/14 and 18/21 both simplify to 6/7

Watch out!

- Order matters: boys : girls is not girls : boys.
- Same units first: 50 cm : 2 m = 50 : 200 = 1 : 4.
- A straight line that misses (0, 0) is **not** proportional.

Kong's tips

- Write labels above the numbers (flour : butter) before you start.
- Ask: should my answer be bigger or smaller? Then check it is.

Level 7–8 challenge

Unfamiliar, multi-step problems: show every step and explain your answer.

Q1 A recipe for 12 gingerbread biscuits uses 300 g of flour and 125 g of butter. The witch needs 4.5 kg of flour to build a wall. How much butter does she need? Show your method.

Check your answer: flour : butter = 300 : 125 = 12 : 5. 4500 ÷ 12 × 5 = 1875 g = 1.875 kg of butter.

Q2 A table has the pairs (2, 7), (4, 14) and (6, 21). Show it is proportional, write its equation, and decide whether (10, 36) is on the same line. Justify.

Check your answer: y ÷ x = 3.5 every time, so y = 3.5x. When x = 10, y = 35, not 36, so (10, 36) is not on the line.

Probability

Probability is a number from 0 (impossible) to 1 (certain) that tells you how likely something is.

Key ideas

- **P(event)** = favourable outcomes ÷ total outcomes
- **Sample space** = every possible outcome. Show it as a list, a table or a tree diagram.
- **Complement:** P(not A) = 1 − P(A)
- **Theoretical** = what should happen. **Experimental** = times it happened ÷ number of trials.
- More trials → experimental gets closer to theoretical.

How to do it

1. **Build the sample space:** a table for 2 events, a tree diagram for 3 or more.
2. **Circle** the outcomes that match the question.
3. **Write** favourable ÷ total, then simplify.
4. **"Not" questions:** work out P(event), then do 1 − P(event).

Worked example

Rock-paper-scissors, two players. P(draw)?

→ 3 × 3 = 9 outcomes. Draws: RR, PP, SS = 3

→ P(draw) = 3/9 = **1/3**

→ P(not a draw) = 1 − 1/3 = **2/3**

Watch out!

- A probability is never above 1 or below 0. Got 7/5? Recount.
- Rock-then-paper and paper-then-rock are two different outcomes.
- Small experiments rarely match theory. That is normal.

Kong's tips

- Two dice? Draw the 6 × 6 grid of totals: 36 outcomes.
- Keep fractions over the same total (like /36) so they are easy to compare.

Level 7–8 challenge

Unfamiliar, multi-step problems: show every step and explain your answer.

Q1 Two dice are rolled and the scores added. Lucas wins on a total of 7 or 11. Mia wins on 2, 3 or 12. Is the game fair? If not, change the rules to make it fair.

Check your answer: Lucas: 6/36 + 2/36 = 8/36. Mia: 1/36 + 2/36 + 1/36 = 4/36. Not fair. One fix: Lucas wins on 7 only (6/36), Mia on 2, 3, 11 or 12 (6/36).

Q2 A spinner has 4 equal sections, one of them red. In 50 spins it lands on red 18 times. Compare the experimental and theoretical probability. Is the spinner unfair? Explain.

Check your answer: Experimental = 18/50 = 0.36. Theoretical = 1/4 = 0.25. 50 spins is too few to be sure: spin many more times, then compare again.

3 ♣

Integers

Integers are whole numbers that can be positive, negative or zero. No fractions, no decimals.

Key ideas

- **Opposites:** 5 and −5. A number + its opposite = 0
- **Absolute value** = distance from 0: |−3| = 3
- **Cartesian plane:** a point is (x, y). x-axis across, y-axis up. 4 quadrants.
- **Order of operations:** Brackets, Indices, × and ÷, then + and − (left to right)

How to do it

1. **Add / subtract:** two signs the same → +, two signs different → −. Then move along the number line. −6 − (−2) = −6 + 2 = −4
2. **Multiply / divide:** count the negatives. Even → positive, odd → negative. (−4) × (−3) = 12
3. **Plot (x, y):** start at (0, 0). Go across by x, then up or down by y.

Worked example

Evaluate $3 - (-2)^2 \times 4$

→ Indices: $(-2)^2 = 4$

→ Multiply: $4 \times 4 = 16$

→ Subtract: $3 - 16 = \mathbf{-13}$

Watch out!

- $(-2)^2 = 4$, but $-2^2 = -4$. Brackets matter!
- −8 is **smaller** than −3. Further left = smaller.
- (2, 5) and (5, 2) are different points. x comes first.

Kong's tips

- Stuck? Sketch a quick number line, or think of a thermometer.
- Tokens: draw + and −, cross out +/− pairs. What is left is the answer.

☆ Level 7–8 challenge

Unfamiliar, multi-step problems: show every step and explain your answer.

Q1 At 6 a.m. it is −8 °C. The temperature rises 3 °C every hour until noon, then falls 2 °C every hour. What is the temperature at 4 p.m.? Show each step.

Check your answer: 6 hours × 3 = +18, so −8 + 18 = 10 °C at noon. 4 hours × (−2) = −8, so 10 − 8 = 2 °C at 4 p.m.

Q2 A(−3, 2), B(4, 2) and C(4, −5) are three corners of a square. Find the fourth corner D and the perimeter of the square.

Check your answer: AB = 4 − (−3) = 7 and BC = 2 − (−5) = 7. D = (−3, −5). Perimeter = 4 × 7 = 28 units.

4 ♥

Expressions & equations

Expressions use letters for numbers. Equations and inequalities let you find what the letter is.

Key ideas

- **Term:** $6x^2y$. The number in front is the **coefficient** (6).
- **Monomial** = 1 term, **binomial** = 2, **trinomial** = 3
- **Degree:** add the powers in a term. A polynomial's degree = its highest term's.
- **Like terms:** same letters, same powers ($3x^2$ and $-5x^2$)
- **Distributive property:** $a(b + c) = ab + ac$

How to do it

1. **Simplify:** collect like terms. $3x^2 - 5x^2 + 4x = -2x^2 + 4x$
2. **Expand:** multiply the outside by every term inside. $-5(3x - 1) = -15x + 5$
3. **Solve:** do the opposite operation to both sides. Undo + and − first, then × and ÷.
4. **Inequality:** same as an equation, but **flip the sign** if you × or ÷ by a negative.

Worked example

Solve $-2y - 7 < 3$

→ Add 7: $-2y < 10$

→ Divide by −2 and flip: **$y > -5$**

→ Number line: open circle at −5, arrow to the right

Watch out!

- The minus outside a bracket multiplies every term inside.
- x and x^2 are not like terms: $2x + 3x^2$ will not simplify.
- Open circle for < and >, filled circle for ≤ and ≥.

Kong's tips

- Check your answer by putting it back into the equation.
- One step per line, with the = signs lined up.

Level 7–8 challenge

Unfamiliar, multi-step problems: show every step and explain your answer.

Q1 Think of a number. Multiply it by 4, add 6, divide by 2, then subtract double your number. Use algebra to prove the answer is always 3.

Check your answer: $n \to 4n \to 4n + 6 \to (4n + 6) \div 2 = 2n + 3 \to 2n + 3 - 2n = 3$, whatever n is.

Q2 A phone plan costs $15 a month plus $0.20 per minute. Maya can spend at most $40. Write and solve an inequality for the number of minutes m she can use.

Check your answer: $15 + 0.2m \le 40 \to 0.2m \le 25 \to m \le 125$. She can use up to 125 minutes.

5 ♦ 2D & 3D geometry

Measure flat shapes (perimeter and area) and solid prisms (surface area and volume).

Key ideas

- **Trapezoid:** $A = \frac{1}{2}(a + b) \times h$ (a, b = parallel sides)
- **Regular polygon:** $A = n \times (s \times a) \div 2$, where a = **apothem** (centre to middle of a side)
- **Circle:** $C = \pi d = 2\pi r$ and $A = \pi r^2$
- **Prism volume:** V = area of base × height
- **Surface area:** add up every face. A **net** shows them all.

How to do it

1. **Regular polygon:** split it into n triangles from the centre. Each one is $s \times a \div 2$.
2. **Surface area:** draw the net. Add the 2 bases and every rectangle round the side.
3. **Volume:** find the base area first, then × the length.
4. **Units:** length in cm, area in cm^2, volume in cm^3.

Worked example

Hexagonal prism: side 4 cm, apothem 3.5 cm, length 10 cm

→ Base = $6 \times 4 \times 3.5 \div 2 = 42$ cm^2

→ V = $42 \times 10 =$ **420 cm^3**

→ SA = $2 \times 42 + 6 \times (4 \times 10) =$ **324 cm^2**

Watch out!

- Use the perpendicular height, never the slanted side.
- Radius or diameter? $r = d \div 2$.
- Surface area has **two** bases. Count both.

Kong's tips

- Write the formula, then substitute. Method marks count!
- Use the π button and round only at the very end.

Level 7–8 challenge

Unfamiliar, multi-step problems: show every step and explain your answer.

Q1 A raised garden bed is a trapezoid prism: parallel sides 6 m and 10 m, 4 m apart, filled with soil 0.3 m deep. Soil comes in 0.5 m^3 bags at $8 each. How much will the soil cost?

Check your answer: Area = $\frac{1}{2}(6 + 10) \times 4 = 32$ m^2. Volume = $32 \times 0.3 = 9.6$ m^3. $9.6 \div 0.5 = 19.2$, so 20 bags × $8 = $160.

Q2 One 30 cm pizza or two 20 cm pizzas cost the same. Which gives you more pizza? Justify with calculations.

Check your answer: $\pi \times 15^2 \approx 706.9$ cm^2. $2 \times \pi \times 10^2 \approx 628.3$ cm^2. The single 30 cm pizza gives more.

6 ♠

Rates

A rate compares two quantities with different units, like kilometres per hour or dollars per kilogram.

Key ideas

- **Ratio** = same units. **Rate** = different units.
- **Unit rate:** the second number is 1 (60 km per 1 hour)
- **Unit price:** the cost of one unit. The smallest one is the best buy.
- **Conversion factor:** e.g. 1 inch = 2.54 cm, 1 mile ≈ 1.6 km
- **Constant rate of change:** straight-line graph, but it does not have to pass through (0, 0)

How to do it

1. **Unit rate:** divide by the second number. 80 miles in 4 h → 20 miles per hour.
2. **Best buy:** price ÷ amount for each option. Smallest wins.
3. **Convert:** multiply by a fraction that cancels the old unit. 26.2 mi × 1.6 km/mi ≈ 41.9 km
4. **Rate of change from a graph:** change in y ÷ change in x.

Worked example

500 g cereal costs $4.00, 750 g costs $5.40

→ $4.00 ÷ 500 = $0.008 per gram

→ $5.40 ÷ 750 = $0.0072 per gram

→ The **750 g box** is the better buy

Watch out!

- Always write units: km/h, $/kg. A bare number loses marks.
- Match units first: minutes → hours, grams → kilograms.
- Multiply or divide? Should the answer be bigger or smaller?

Kong's tips

- Keep units in every line of working. They tell you what to do next.
- Comparing per 100 g often gives friendlier numbers.

Level 7–8 challenge

Unfamiliar, multi-step problems: show every step and explain your answer.

Q1 Car A uses 6 L of fuel per 100 km. Car B travels 15 km per litre. Which car is more fuel-efficient? How much would a 450 km trip cost in Car A if fuel is $1.80 per litre?

Check your answer: Car A: 100 ÷ 6 ≈ 16.7 km per litre, more than 15, so Car A. Fuel = 4.5 × 6 = 27 L → 27 × $1.80 = $48.60.

Q2 Jeans cost $50 in the USA or €48 in France. The exchange rate is 1 USD = 0.90 EUR. Where are they cheaper, and by how much?

Check your answer: $50 × 0.90 = €45. They are cheaper in the USA, by €3.

7 ♣

Univariate data

Univariate data is about one thing, like everyone's test score. Describe its middle and its spread.

Key ideas

- **Mean** = total ÷ how many. **Median** = middle value in order. **Mode** = most common.
- **Range** = largest − smallest
- **Q1** = middle of the lower half, **Q3** = middle of the upper half. **IQR** = Q3 − Q1
- **Five-number summary:** min, Q1, median, Q3, max → draw a **box-and-whisker plot**
- **Stem-and-leaf plot** keeps every value. Always add a key: 8 | 3 = 83

How to do it

1. **Order the data first.** Every other step depends on it.
2. **Median:** cross off one value from each end. Even count → average the middle two.
3. **Q1 and Q3:** split the data at the median, then find the middle of each half.
4. **Outlier?** Use the median to describe the data, not the mean.

Worked example

Data: 3, 5, 7, 8, 8, 10, 12, 15

→ Mean = 68 ÷ 8 = **8.5** · median = **8** · mode = **8**

→ Range = 15 − 3 = **12**

→ Q1 = 6, Q3 = 11, IQR = **5**

Watch out!

- Data not in order = wrong median and quartiles.
- The range is one number (12), not "3 to 15".
- A stem-and-leaf plot with no key loses marks.

Some tips

- Count your values before and after ordering. Same number?
- Comparing box plots: look at the medians (centre) AND the IQRs (spread).

☆ Level 7–8 challenge

Unfamiliar, multi-step problems: show every step and explain your answer.

Q1 Homework minutes for 7 students: 4, 5, 5, 6, 7, 8, 40. Find the mean and the median. Which describes a typical student better? Explain why.

Check your answer: Mean = 75 ÷ 7 ≈ 10.7 min. Median = 6 min. The median is better: 40 is an outlier that pulls the mean up.

Q2 Class A: min 40, Q1 55, median 65, Q3 75, max 90. Class B: min 50, Q1 60, median 64, Q3 68, max 80. Compare the two classes' test scores.

Check your answer: Medians are almost equal (65 vs 64), so the centres are similar. Class B is more consistent: IQR 8 vs 20, range 30 vs 50.