1 Number

Rational numbers, exponent laws and scientific notation help you work with very big and very small numbers.

Key ideas

- **Rational number:** can be written as a fraction of two integers (bottom ≠ 0). **Irrational** numbers, like $\sqrt{2}$ and π, cannot.
- **Product rule:** $a^m \times a^n = a^{m+n}$. **Quotient rule:** $a^m \div a^n = a^{m-n}$
- **Power of a power:** $(a^m)^n = a^{mn}$. Same power: $a^n \times b^n = (ab)^n$
- **Zero and negative powers:** $a^0 = 1$, $a^{-n} = 1/a^n$. **Fractional:** $x^{1/n} = \sqrt[n]{x}$
- **Scientific notation:** $a \times 10^n$, where $1 \le a < 10$

How to do it

1. **Finite decimal → fraction:** read the place value. 0.324 = 324/1000 = 81/250
2. **Recurring decimal → fraction:** call it x, multiply by 10s until the decimal parts match, subtract, solve.
3. **× or ÷ in scientific notation:** do the numbers and the powers of 10 separately, then tidy so $1 \le a < 10$.
4. **+ or − in scientific notation:** rewrite with the same power of 10 first, then add or subtract.

Worked example

Write 0.353535... as a fraction

→ Let x = 0.3535..., so 100x = 35.3535...

→ Subtract: 99x = 35

→ x = **35/99**

Watch out!

- $a^0 = 1$, not 0.
- 22.5×10^7 is not scientific notation: write 2.25×10^8.
- A negative power is not a negative number: $2^{-3} = 1/8$.

Kong's tips

- Learn the laws as a family: × adds powers, ÷ subtracts, brackets multiply.
- Practise with your calculator's $\times 10^x$ (EXP) button before the test.

Level 7–8 challenge

Unfamiliar, multi-step problems: show every step and explain your answer.

Q1 Light travels at 3.0×10^8 m/s. The Sun is 1.5×10^{11} m from Earth. How long does sunlight take to reach Earth? Give your answer in seconds and in minutes.

Check your answer: $(1.5 \div 3.0) \times 10^{11-8} = 0.5 \times 10^3 = 5 \times 10^2$ s = 500 s ≈ 8.3 minutes.

Q2 Simplify $(2x^3y^{-2})^3 \div (4x^5y^{-4})$. Write your answer using positive powers only.

Check your answer: $(2x^3y^{-2})^3 = 8x^9y^{-6}$. Dividing: $2x^4y^{-2} = 2x^4/y^2$.

2 ♠

Triangles

Use Pythagoras' theorem, similar triangles and trigonometry to find missing sides and angles.

Key ideas

- **Pythagoras:** $c^2 = a^2 + b^2$, where c is the **hypotenuse** (opposite the right angle). Right triangles only!
- **Congruent:** an exact copy. Same sides, same angles.
- **Similar:** an enlargement. Same angles, sides in proportion.
- **Similar if:** AA (2 angles match), SAS or SSS (sides in proportion)
- **SOH CAH TOA:** sin = opp/hyp, cos = adj/hyp, tan = opp/adj

How to do it

1. **Missing side (Pythagoras):** square, add (or subtract for a shorter side), then square-root.
2. **Similar triangles:** match corresponding sides, find the scale factor, multiply or divide.
3. **Trig side:** label opp, adj, hyp from the angle. Choose SOH, CAH or TOA. Solve.
4. **Trig angle:** use the inverse button: $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$.

Worked example

Right triangle: angle 35°, hypotenuse 12 cm. Find the opposite side.

→ opp and hyp → SOH: $\sin 35° = x/12$

→ $x = 12 \times \sin 35°$

→ $x \approx$ **6.88 cm**

Watch out!

- Calculator in degrees (DEG), not radians.
- The hypotenuse is always the longest side.
- Similar is not congruent: similar triangles can be different sizes.

Kong's tips

- Always sketch and label the triangle, even if one is given.
- Found a hypotenuse? Check it is the biggest side.

Level 7–8 challenge

Unfamiliar, multi-step problems: show every step and explain your answer.

Q1 A 6 m ladder leans against a wall with its foot 1.5 m from the wall. How high up the wall does it reach, and what angle does it make with the ground?

Check your answer: $h = \sqrt{(6^2 - 1.5^2)} = \sqrt{33.75} \approx 5.81$ m. $\cos\theta = 1.5/6 = 0.25$, so $\theta = \cos^{-1}(0.25) \approx 75.5°$.

Q2 A 1.6 m student casts a 2 m shadow. At the same moment, a tree casts a 15 m shadow. How tall is the tree? Explain why the triangles are similar.

Check your answer: Similar by AA: both have a right angle and the same sun angle. $1.6/2 = h/15$, so $h = 12$ m.

3 ♣ Linear relationships

A linear relationship makes a straight line. Its gradient and intercepts tell its story.

Key ideas

- **Gradient:** m = rise ÷ run = $(y_2 - y_1) \div (x_2 - x_1)$. Positive slopes up, negative slopes down.
- **Parallel lines:** same gradient. **Perpendicular:** gradients multiply to −1 (2 and −½).
- **Intercepts:** put x = 0 to find the y-intercept, y = 0 to find the x-intercept.
- **Three forms:** $y = mx + c$ · $y - y_1 = m(x - x_1)$ · $Ax + By = C$
- **Horizontal** line y = a (gradient 0). **Vertical** line x = a (gradient undefined).

How to do it

1. **Graph y = mx + c:** plot c on the y-axis, use rise/run to find more points, join with a ruler.
2. **Equation from two points:** find m, then put m and one point into $y - y_1 = m(x - x_1)$.
3. **Intercepts from Ax + By = C:** set x = 0 to get y, then set y = 0 to get x.

Worked example

Find the line through (1, 3) and (4, 9)

→ m = (9 − 3) ÷ (4 − 1) = 2

→ y − 3 = 2(x − 1)

→ **y = 2x + 1**, y-intercept (0, 1)

Watch out!

- Subtract in the same order on top and bottom of the gradient.
- A vertical line's gradient is undefined, not 0.
- Intercepts are points: write (0, 1), not just 1.

Kong's tips

- In real life, the y-intercept is the starting value and the gradient is the rate.
- Check your equation using the second point.

Level 7–8 challenge

Unfamiliar, multi-step problems: show every step and explain your answer.

Q1 A taxi charges a fixed fee plus a cost per km. A 4 km ride costs $11 and a 10 km ride costs $20. Find the equation and explain what the gradient and y-intercept mean.

Check your answer: m = (20 − 11) ÷ (10 − 4) = 1.5. 11 = 1.5(4) + c, so c = 5. C = 1.5d + 5: $1.50 per km plus a $5 fixed fee.

Q2 Line L is 3x − 2y = 12. Find its intercepts, then the equation of the line perpendicular to L that passes through (0, 4).

Check your answer: Intercepts (4, 0) and (0, −6). L is y = 1.5x − 6, so m = 3/2. Perpendicular m = −2/3: y = −2/3 x + 4.

3D shapes

Find the surface area and volume of cylinders, pyramids and spheres.

Key ideas

- **Prism or cylinder:** V = area of base × height
- **Cylinder:** V = $\pi r^2 h$ and SA = $2\pi r^2 + 2\pi rh$
- **Pyramid:** V = ⅓ × base area × height (⅓ of the prism with the same base and height)
- **Pyramid surface area:** base + every triangle face. Use the net and the **slant height**.
- **Sphere:** V = 4/3 πr^3 and SA = $4\pi r^2$

How to do it

1. **Name the shape** and write its formula first.
2. **Get the right measurements:** radius, not diameter. Perpendicular height for volume, slant height for triangle faces.
3. **Substitute and calculate.** Round only at the end.
4. **Units:** area in cm^2, volume in cm^3.

Worked example

Cylinder: r = 3 cm, h = 10 cm

→ V = $\pi \times 3^2 \times 10 = 90\pi \approx$ **282.7 cm^3**

→ SA = $2\pi(3^2) + 2\pi(3)(10) = 18\pi + 60\pi$

→ SA = $78\pi \approx$ **245.0 cm^2**

Watch out!

- Pyramids: slant height for surface area, perpendicular height for volume.
- Given the diameter? Halve it first.
- Don't forget the ⅓ for pyramids.

Kong's tips

- Sketch the net so you count every face once.
- Keep answers in terms of π until the last step.

Level 7–8 challenge

Unfamiliar, multi-step problems: show every step and explain your answer.

Q1 A cylindrical can holds 3 tennis balls of diameter 6.5 cm that fit exactly (touching the sides, top and bottom). What percentage of the can is empty space?

Check your answer: Can: r = 3.25, h = 19.5 → V = 205.97π. Balls: $3 \times 4/3\ \pi(3.25)^3 = 137.31\pi$. Balls fill 2/3, so ≈ 33.3% is empty.

Q2 A square pyramid has a base of 10 m and a height of 12 m. Find its slant height, its volume and its total surface area.

Check your answer: Slant = $\sqrt{(12^2 + 5^2)} = 13$ m. V = ⅓ × 100 × 12 = 400 m^3. SA = 100 + 4 × ½ × 10 × 13 = 360 m^2.

Bivariate data

Bivariate data compares two variables to find out whether, and how, they are related.

Key ideas

- **Scatter plot:** one point for each pair of values.
- **Correlation:** positive (both rise), negative (one rises, one falls) or none.
- **Strength:** weak, moderate or strong. The **correlation coefficient r** runs from −1 to 1.
- **Line of best fit** (trend line): follows the pattern and passes through the mean point $(\bar{x}, \bar{y})$.
- **Correlation does not mean causation.**

How to do it

1. **Plot** each pair, with the independent variable on the x-axis.
2. **Describe:** direction + strength + any outliers, in context.
3. **Line of best fit:** plot the mean point, draw through it following the trend, with points on both sides.
4. **Predict** by reading from the line or its equation. A GDC gives r and the equation.

Worked example

What does each value of r tell you?

→ r = 0.92 → **strong positive**

→ r = −0.45 → **moderate negative**

→ r = 0.05 → **no linear relationship**

Watch out!

- Don't join the dots or force the line through (0, 0).
- Predicting far outside the data range is unreliable.
- A strong r never proves one thing causes the other.

Kong's tips

- Use the full sentence: "There is a strong positive correlation between... and..."
- Check both axis scales before you plot.

Level 7–8 challenge

Unfamiliar, multi-step problems: show every step and explain your answer.

Q1 Hours studied vs test score: (1, 52), (2, 58), (3, 61), (4, 70), (5, 74). Find the mean point, draw a line of best fit and predict the score for 6 hours. Is a prediction for 20 hours sensible?

Check your answer: Mean point (3, 63). Best-fit line ≈ y = 5.6x + 46.2, so 6 hours ≈ 80. For 20 hours it predicts 158, more than 100%: unreliable.

Q2 A study finds a strong positive correlation between the number of firefighters at a fire and the damage caused. Do firefighters cause damage? Explain.

Check your answer: No. Correlation is not causation: bigger fires bring more firefighters and also cause more damage (a third variable).

6 ♠

Transformations

Transformations move, flip, turn or resize a shape. The original is the preimage; the result is the image.

Key ideas

- **Congruence** (same size): translation, reflection, rotation. **Similarity** (new size): dilation.
- **Translation:** $(x, y) \to (x + h, y + k)$
- **Reflection:** x-axis $(x, -y)$ · y-axis $(-x, y)$ · $y = x$ (y, x) · $y = -x$ $(-y, -x)$
- **Rotation about (0, 0):** 90° anticlockwise $(-y, x)$ · 180° $(-x, -y)$ · 90° clockwise $(y, -x)$
- **Dilation**, centre (0, 0), scale factor a: $(x, y) \to (ax, ay)$
- **Tessellation:** shapes cover a surface with no gaps and no overlaps.

How to do it

1. **Write the rule**, then apply it to every vertex.
2. **Plot and label** the image A', B', C'.
3. **Dilation:** factor > 1 enlarges, between 0 and 1 shrinks, negative flips to the other side of the centre.
4. **Describe fully:** the type + its details (vector, mirror line, angle and direction, or scale factor and centre).

Worked example

A(1, 2), B(4, 2), C(1, 5): rotate 90° anticlockwise about (0, 0)

→ Rule: $(x, y) \to (-y, x)$

→ A'(−2, 1), B'(−2, 4), C'(−5, 1)

Watch out!

- Always say clockwise or anticlockwise for a rotation.
- Reflecting in $y = x$ swaps the coordinates; in the y-axis only x changes sign.
- A dilation changes size, so the image is similar, not congruent.

Kong's tips

- Use tracing paper for rotations: pin the centre and turn.
- Test the rule on one vertex before doing the rest.

Level 7–8 challenge

Unfamiliar, multi-step problems: show every step and explain your answer.

Q1 Triangle P(2, 1), Q(5, 1), R(5, 3) is reflected in the y-axis, then translated 3 units down. Find the final coordinates. Would the opposite order give the same result? Explain.

Check your answer: Reflect: (−2, 1), (−5, 1), (−5, 3). Down 3: (−2, −2), (−5, −2), (−5, 0). Same result: the reflection changes only x, the translation only y.

Q2 A square with vertices (1, 1), (3, 1), (3, 3), (1, 3) is dilated from the origin by a scale factor of 3. Find the new vertices. How many times bigger are the perimeter and area?

Check your answer: (3, 3), (9, 3), (9, 9), (3, 9). Perimeter × 3 (8 → 24). Area × 9 (4 → 36).

7 ♣

Linear systems

A linear system is two linear equations solved together. The solution makes both equations true.

Key ideas

- **Degree:** linear 1, quadratic 2, cubic 3. An equation usually has as many solutions as its degree.
- **Two unknowns** need at least two equations.
- **Graphing:** the solution is where the lines cross.
- **Substitution:** get one variable alone, put it into the other equation. **Elimination:** make coefficients opposite, then add.
- **No solution:** parallel lines (you get something false, like 0 = 2). **Infinitely many:** the same line (0 = 0).

How to do it

1. **Choose a method:** substitution if a letter is already alone (y = ...), elimination otherwise.
2. **Solve** for one variable.
3. **Substitute back** to find the other.
4. **Verify** your pair in BOTH original equations.

Worked example

Solve 2x + 3y = 12 and x − y = 1

→ From the 2nd: x = y + 1

→ 2(y + 1) + 3y = 12 → 5y = 10 → y = 2

→ x = 3. Solution **(3, 2)**

Watch out!

- When you multiply an equation, multiply EVERY term.
- Find both values: one number is only half an answer.
- 0 = 0 means infinitely many solutions, not none.

Kong's tips

- Write the answer as a point: (3, 2).
- Word problems: define your letters first (let a = adult tickets).

Level 7–8 challenge

Unfamiliar, multi-step problems: show every step and explain your answer.

Q1 A school sells 200 tickets. Adult tickets cost $8 and student tickets cost $5. The total is $1300. How many of each ticket were sold? Verify your answer.

Check your answer: a + s = 200 and 8a + 5s = 1300. s = 200 − a → 3a + 1000 = 1300 → a = 100, s = 100. Check: 800 + 500 = 1300.

Q2 Show that y = 2x + 3 and 4x − 2y = 5 have no solution. What does this mean on a graph?

Check your answer: 4x − 2(2x + 3) = 5 gives −6 = 5, which is false. Both lines have gradient 2 but different intercepts: they are parallel and never meet.